FDU: A CASE STUDY OF THE PROCESS OF HIGHER EDUCATION[1]

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A. Introduction

Social scientists have traditionally looked outside of their own community for problems requiring rational analysis and solutions. As educators, they have tried to enlighten their students on the quantitative properties of processes taking place outside of their own classrooms. However, until recently, the quantitative properties of one of their most important activities, the educational process itself, have remained outside the social scientists' purview. While refusing to heed intuition in other areas of human endeavor, social scientists have tended to rely on guesswork as a basis for decision-making within their own classrooms.

Indeed, education is a major area of public decision-making and concern. Respectable theories have been built using education as one of the main components of human capital, and thus as one of the main determinants of private income and wealth.[2] The possibility of affecting the income and wealth distribution of the population by means of rational educational policies thus becomes clear and desirable. Nevertheless, educational levels cannot be controlled unless the learning process is clearly understood.

In the last few years, economists have produced a large and evergrowing, collection of "production-function" studies. These studies have tried to establish how resources are allegedly used in the attainment of alternative levels of production. Functions having a priori desirable properties, such as the C-D and CES production functions have been called upon to link inputs and outputs in specified ways. However, it is well known that the power of these hypotheses is small; alternatively highly probable explanations usually offer conflicting views of the empirical findings. In the field which concerns us now, input-output relations, having uncertain empirical properties, have multiplied in number -- leaving the educational researchers with the same unanswered questions that baffled their methodological predecessors.

Other social scientists, having developed no vested interests in the empirical viability of partial technological relations, have tried to explain the process of education in terms of socio-economic determinants. Unfortunately, the latter type of studies have tended to disregard the a priori properties of their postulations -- increasing the likelihood of confounding.

In this paper, I will develop a theory of educational decision-making, and summarize its empirical implications upon a specific sample of data on higher educational variables, drawn from Fairleigh Dickinson University's student population.[3] The theoretical approach will focus upon the implications of constrained decision-making. The observed differences in student achievements, or educational outcomes, will not only be connected with technological and endowment differences, as it is true of the traditional "production-function" studies, but also with aspirations, or goal variations. However, the interactions between goal and technological relations will be established iteratively, in order to determine the separate impact of each of the two sets of variables respectively. The empirical approach will be Bayesian in nature, in the sense that the data will be used to infer the theoretical classifications. In other words, rather than "explain" achievement levels, I will use the observed levels of achievement in order to determine the likelihood of the postulated theory.

B. An Exact Decision-Making Theory

1. The empirical manifestations of the relations existing between educational inputs and outputs are not independent from the goals and objectives of the individuals and institutions involved. It is only commonsensical to link student achievement to own evaluations of education -- or to the educational policies of the colleges in question. Thus, unless the crucial role played by goals and objectives is clearly recognized at the outset, any predictive statement of student behavior, failing to take it explicitly into account, will be subject to an uncertain degree of bias.

2. In a restricted sense, given our ignorance of the student's spectrum of goals, resources and technological constraints, we could linearly approximate the decision-making problem of the student as follows:

$$\max U = u'x \text{ s.t. } (I\ Z : IIx \mid \overset{x}{\pm s}) = b, \text{ where}$$

u'= vector of individual, ordinal preference weights with respect to the various x, U= scalar representing level of goal attainment, x= vector of outputs -- educational, recreational, work, or goals of the individual student, Z= matrix of educational, recreational, work technology coefficients, I= identity matrix, b= vector of given resource constraints, s= vector of slack variables associated with the vector of endowments b.

Consider Z_B as the optimal basis for the problem at hand, such that x*, c* are optimal if and only if $x^*_j=0$ for $z'_j c^* > u_j$, and $c^*_i=0$ for $z_i x^* < b_i$.[4] These conditions imply that students will neither choose to produce any output x^*_j for which its ordinal returns per unit -- the price of x^*_j, u_j -- are smaller than its imputed costs, $Z_j c^*$, nor consider any resource which remains unused valuable.

At the optimum, which is an extremum, or a set of extrema, $Z_B x^* = b$ or $Z_B^{-1}(b) = x^*$. Thus, for any output x*=

$(Z_B^{-1}) x\ b$, and $x^*>0$ if and only if

$\sum_j (Z_{1j} c_j) - \mu_1 < 0$ -- where the subscript $_{1j}$

indicates the matrix element in row 1 and column j. These necessary and sufficient "marginal" conditions could also be restated as follows:

$(Z_B)_j = (\mu_i / c_j)$, j= 1, 111, n.

If we let $\overset{n}{x}{}^*_1$ stand for a particular educational output, and b for a column vector whose elements are the levels of a chosen set of educational resources, we may state the strong hypothesis that educational outputs are a function of educational inputs. This is a strong hypothesis because we are assuming both a specific linear form for our approximation and a given evaluation of educational outcomes. Moreover, assuming that students decisions approach optimality, we will conclude that those resources which do not appear to be used in the achievement of a particular educational goal have a negative yield -- returns on their use are less than their costs.

Take, for instance, cumulative grade averages. Students having a given level of intelligence, faculty assistance, library use, etc. will achieve, according to our postulation, the same grade average if their evaluation of different grade averages is consistent as well. It is worth emphasizing that two students with the same resources may achieve different grade levels because their goals are dissimilar.

C. Statistical Methodology: Parameter Estimation

1. Our methodology will be perfectly general, and thus applicable to any perceived educational output. Since our preliminary empirical tests concentrate on cumulative grade averages, we will phrase our statistical considerations in terms of those variables -- without loss of generality and for ease of understanding. According to our hypothesis, student cumulative grade averages, as an educational output, may be expressed as a linear combination of student resources, provided the subjective, relative evaluation of the importance of grades is maintained constant within each grade group. Empirically, this hypothesis implies that we should expect students in different cumulative grade average classes to exhibit, respectively, characteristic patterns of resource endowments and use. For each grade level x_1 -- F, D, C, B, A, -- we will compute a vector $(Z^{-1})_{1.}$ such that the resulting sample scalar $(Z^{-1})_{1.} b_{.a} = x_{1a}$ for each of the students, a= 1, 2, ..., n_1, in that grade class will minimize the probability of classifying that student, a, in a grade level other than that in which he is observed. $b_{.a}$ is an (mxl) column vector of m resource levels for student a. In other words, given several groups of students exhibiting grade averages ranging from F to A, we want to determine a set of constants for each group such that the corresponding linear combinations of student resources will yield for each group respectively a range of scalars which minimize the probability of misclassification.

2. Consider the following matrix of observations: $B = /b_{ij}/$, with typical element b_{ij}, where i= resource variable under considerations, i= 1, 2, ..., r, and j= ordered student number, j= 1, 2, ..., $\sum_{1=1}^{p} n_1$. Order the columns of B in such a way that the first n_1 correspond to the lowest grade students, n_2 to the next to the lowest grade, etc. up to the last n_p observations -- corresponding to the highest cumulative average grade students. Define S=

$$(BB' - \sum_{1=1}^{p} (n_1 \bar{b}\bar{b}')/((\sum_{1=1}^{p} n_1) - p),$$

where $\bar{b}(\bar{b}_1 \bar{b}_2 \bar{b}_3 \ldots \bar{b}_m)'$, and $\bar{b}_i =$

$(b_{i.} xI)/\sum_{1=1}^{p} n_1$ -- I is an $(\sum_{1=1}^{p} n_1)$xl column vector of ones -- as the unbiased or pooled covariance matrix for the whole sample.

Then, for each grade group, the maximum likelihood estimates of $(Z^{-1})_{1.}$ will be as follows:

$(Z^{-1})_{1.}=((-\frac{1}{2}\bar{b}_{.1}'S^{-1}\bar{b}_{.1})\ (\bar{b}_{.1}'S^{-1}))$,[5]

where $\bar{b}.= (b_1'\ I/\ n_1)\ (b_2'I/n_1)\ \ldots$

$(b_m/I/n_1)$ and I is a $(\sum_{1=1}^{p} n_1x1)$ vector,

with ones corresponding to students who got grades x_1, and with zeroes elsewhere. It is worth reminding the reader that $(Z^{-1}_{1.}) - (Z^{-1})_{k.}=D_{1k}$ for $k \neq 1$ represents the discriminant function coefficients between groups 1 and k. In this particular case, we have a total of $(p^2-p)/2$ discriminant functions.

3. Some observations will appear to be inconsistent, in the sense that, according to resources, students will be classified in grade groups different from those in which they were actually observed. We may infer, from our previous analysis, that such inconsistencies are due to goals differences. We strongly hypothesize that, for instance, all A students observed classified as B, C, D, or F, according to their resource endowments, attained the highest grade as a consequence of their different evaluation of grade outputs. In order to identify such differences, we will associate goal levels with a relevant set of socio-psychological background variables. In symbols, we will consider the following linear approximation:

$u^a_{11}*= K.g^a$ where $u^a_{11}*=$ value of cumulative grade average to student a, who attained a level x_1* but appeared, in terms of resources, to belong to the group x_1, 1=A, B, C, D, F, K=(lxm) vector of marginal, constant goal contributions, m= total number of socio-psychological background variables, p= total number of grade groups, $g^a=$ (mxl) vector of socio-psychological background variable levels corresponding to student a.

In passing, it may be noted that an ordinal notion of the marginal costs of the resources employed by students in each grade classification may be gained from the following relation:

$(Z^{-1})'_1*u'_{11}*=c_{.1}*$, which defines, given the programming equilibrium conditions, the implied cost of such resource endowment, at the margin, for a student attaining a grade level 1*, but belonging to the resource group 1. In general, since $K.g= u_{11*}$, we may state that $c_{.1}*= ((Z'^{-1})_{1*}\ K.g)$, for each 1, 1*, K triplet applicable.

4. In sum, the statistical methodology which will be applied, in order to identify endowment and goal influences at each cumulative grade level may be summarized as follows:

(a) Classify students according to cumulative grade levels.
(b) Estimate a set of "marginal product" coefficients for each group, in order to explain grade differences in terms of resource endowments and utilization.
(c) Estimate a set of "marginal goal contribution" coefficients for each group of similarly misclassified students within each originally observed grade, in order to explain switches in terms of socio-psychological background variables.

5. In the first iteration, once the inputs considered of importance have been isolated for each grade group, it becomes important to determine which variables appear to be instrumental in promoting students to cumulative grade averages other than that which they have attained. Given five grade levels, techniques of producing grades, there are ten possible distinct movements we may examine. In other words, we may ask the following types of questions: if, according to observed results, a student attained a B average, and he wants to promote his grade to the A level, should he increase hours of library study, and/or cut down part time work activities, and/or seek enlarged faculty assistance, etc.? Similarly, in the second state or iteration, we want to determine those socio-psychological background variables which appear to induce grade-attainment inconsistencies.

Statistically, those marginal "expansion" coefficients can be defined as: $S^{-1}(\bar{b}_{.1}-\bar{b}_{.k})=((Z^{-1})_{.1}-(Z^{-1}{}_{.k}))$ for all groups, $1\neq k$, and upon pairwise comparison.[6]

D. Statistical Methodology: Sample Distribution

1. Total Distance

Whether the optimal classification is statistically significant, or just a "figment of sampling variation" may be determined in terms of the distance between the mean vectors of the optimal groupings and that of the total sample. P. C. Mahalanobis'[7] generalized distance measure may be applied, and its significance determined, for samples of the size we will consider. For example, for the first iteration, on educational resources,

$$M=\sum_{l=A}^{F}(\bar{b}_1-\bar{\bar{b}})' S^{-1}(\bar{b}_1-\bar{\bar{b}})$$ where M= generalized Mahalanobis distance, $\bar{b}$= vector of resource means for grade level 1, $\bar{\bar{b}}$= $(\bar{b}'i/p)$, p= total number of resource groups -- as the reader may recall from the previous discussion -- i= a vector of ones (lxr), and r= number of resource constraints at the optimum. M has a χ^2 distribution with px(r-1) degrees of freedom.

2. Intergroup Distances

Testing whether the groups' mean vectors of resources are significantly different from the sample's total mean resource vector or not gives us a general idea of the statistical validity of our hypothesis. However, we may want to test the statistical significance of the (px(p-1)/2) distances among the mean resource vectors, upon pairwise comparison, of the p groups. In this regard, we may recall that

$$T^{*2}=(\bar{b}_1-\bar{b}_k)'S^{-1}(\bar{b}_1-\bar{b}_k).\ ((n_1+n_k)-1)_{1\neq k},$$

where T^{*2}= Hotelling's T^2, which transforms, upon multiplication by $(n_1+n_k-r)/r(n_1+n_k-1)$ into a central F^8 variable with r and (n_1+n_k-r) degrees of freedom, and $\bar{b}_1$ and $\bar{b}_k$ represent groups' l and k mean resource vectors respectively.

Thus, $F_{1k}=2([(Z^{-1})_{.1}-(Z^{-1})_{.k}]'\bar{b}_1)$. $(n_1+n_k-r)/r$ and the (px(p-1)/2) hypothesis may be tested at any desired level of significance respectively upon comparison of F_{1k} and F^{α}, where F^{α} represents a critical calue for a significance region of size α.

3. Posterior Probabilities

The probability that a given student will be considered well-classified or not may be determined in a "Bayesian" fashion. For instance, consider the first discrimination iteration, in terms of resource endowments. In particular, for a student a, who achieved an x_2 average, five values, x_1= A, B, ... F, $(Z^{-1})_{1.}$ x x $b_{.a}$= f_{x1}^{a} may be computed, and the maximum f^a_{x1}= f^M determined. Then, we may establish a priori, that $P(x_2/x_1).P(x_1)= g(f^M - f^a_{x1})$, choosing a function g which will achieve a maximum at g(0), and such that $0\leq g<1$. The exponential $\exp(-(f^M - f^a x_1))$, for instance, fulfills our requirements. Thus, the posterior $P(x_1/x_2)$, for l= 1, ..., 5, may then be computed using Bayes' theorem.

Alternatively, since $(f^a_{x1}-f^a_{xk})$= $N((\bar{f}_{x1}-\bar{f}_{xk}),\ 2(\bar{f}_{x1}-\bar{f}_{xk}))$, $1\neq k$, $\bar{f}_{xk}$= $((Z^{-1})k).\ x\bar{b}_{.k}$, $\bar{f}_{x1}$= $(Z^{-1}{}_{1.})$ x $\bar{b}_{.k}$ if x_k is true, and $(f^a_{xk} - f^a_{x1})= N(-\bar{f}_{xk}+\bar{f}_{x1}),$ $2(\bar{f}_{xk} - \bar{f}_{x1}))$, $k\neq Z$, when x_1 is true, pairwise critical regions may be established for each student vector.

4. Dependence and Prediction

Whether the postulated hypothesis: (1) patterns of educational resource use and outcomes are functionally related, and (2) inconsistencies of resource use and outcomes are due to educational goal differences, are empirically corroborated or not may be tested in alternative ways.

Contingency tables indicating resource classification frequencies for each alternative outcome, average cum grades, and goal classification frequencies for each alternative resource claslification within a particular outcome group may be constructed. Then, Pearson's χ^2 approximation may be used to test the independence of (1) specific, or optimal patterns of resource use from their corresponding outcomes, and of (2) specific, or optimal patterns of social background variables from that of alternative resource use classification within a particular educational outcome group.

If resource use patterns can be used to predict educational outcomes, a significantly positive correlation between typical resource patterns and observed outcome classifications will exist. Analogously, given any specific educational outcome level, say average cum grades, resource use levels inconsistent with those typical for the average cum in question will have to be explained by systematic differences in social background or goal variable patterns. Thus, similarly "inconsistent" resource vectors will be positively correlated with social-background variable patterns. The Student-t distribution may be used to determine the significance of the sample correlation coefficients between educational outcomes and resource-use classifications, as well as between resource-use and social background, or goal classification within a particular educational outcome group.

The relative frequency of accurate prediction of attained levels of educational achievement may be used as an indication of the explanatory ability of the postulated hypotheses. I call this ratio the consistency coefficient $E = \sum_{ij}(f_{ii}/\sum_{ij}f_{ij})$, where f_{ij}= number of students attaining an original classification level i, and whose discrimination, or optimal new classification level is j.

E. Summary of Empirical Findings

Our analysis of the Fairleigh Dickinson University questionnaire evidence leads to the following empirical statements:

(1) Statistically significant, distinctive educational resource use vectors characterize student average-cum grade achievements.

(2) Statistically significant, distinctive social background or goal vectors characterize specific differences between student educational resource use patterns and their corresponding average-cum grade achievements.

(3) The optimal patterns of educational resource use and average-cum grade achievements are dependent and positively correlated. Both, dependence and correlation are statistically significant.

(4) The optimal patterns of educational background and resourse use variables, within particular average-cum groups of students, are dependent and positively correlated. Both, dependence and correlation are statistically significant.

(5) The explanatory ability with respect to student average-cum grades of educational resource use and social back-ground patterns ranges from a third to more than one half of all observations.

(6) At different average-cum grade levels, definite patterns of resource use and goal variable substitution are observable. Specific grade levels may be attained in a number of resource use and social background or goal variable level combinations.

(7) The analytical methodology developed appears to be robust, in the sense that reversing the order of iteration, or changing the number of components in the explanatory vectors will not substantially alter the empirical conclusions.

Some qualifications to our findings are certainly in order. The general validity of our quantitative parameter estimates may be impaired by the nature of the sample considered. The biases introduced by probable self-selectivity or respondents, subjective content of graphic forced-choice questions, imperfect proxies, restrictiveness of sample, etc., though unknown, should not be ignored. Clearly, further testing of the theory on more comprehensive bodies of data will serve the purpose of generalizing our preliminary research. However, the empirical implications of our conclusions are strong as well as eminently practical: the process of education and educational achievement can be quantitatively assessed and the resulting parameter estimates used to devise efficient plans for university, student, and faculty resource utilization.

Footnotes

1. This paper was written under an F.D.U. Faculty Grant. I would like to specially acknowledge the advice of Dr. A. Jaffe, the data processing help of Mrs. Louise Yanoff, and the expert typing of Mrs. Marilyn Meyers.

2. Among the very many references, two are worth mentioning; Friedman, M., "Choice, Chance, and the Personal Distribution of Income," The Journal of Political Economy, vol. vli, #4, 1953, pp. 277-9-, and Becker, G.S., Human Capital, New York, 1964.

3. Lack of space prevents a full reporting of my preliminary findings. However, I will be happy to supply the interested reader with detailed statistical results.

4. C is a vector of imputed costs of resources; this is the so-called equilibrium theorem of linear programming. See, for instance, Lancaster, K. J., Mathematical Economics, New York, 1968, pp. 33-34.

5. See, for instance, Anderson, T. W.,

An Introduction to Multivariate Statistical Analysis, Chapter 6, New York, 1958.

6. For a development of this discrimination concept see, for instance, Hoel, P. G., Introduction to Mathematical Statistics, Third Edition, New York, 1964, pp. 179ff.

7. See Mahalanobis, P. C., "On the Generalized Distance in Statistics," Proceedings of the National Institute of Sciences, Vol. xii, Calcutta, India, 1936, pp. 49-55.

8. For a derivation of the distribution of Hotelling's T^2, see Anderson, T. W., Op.cit.